International School of Multiscale Mathematical Models for Multi-Agent Systems

Multiscale Mathematical Models for Multi-Agents Systems

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Book of Abstracts

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List of Speakers

A physical-mathematical review of pilot-wave models

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Pilot-wave theories provide a realist, trajectory-based framework for quantum dynamics in which particles evolve under the guidance of an associated wave field. Originating from de Broglie's double-solution program and later reformulated in Bohmian mechanics, these models have undergone significant conceptual and mathematical development, particularly in connection with modern hydrodynamic analogues and field-theoretic approaches. This talk presents a physical-mathematical review of pilot-wave models, with emphasis on their underlying equations, coupling structures, and limiting regimes. I will contrast the single-wave formulation of Bohmian mechanics with de Broglie's original double-solution idea, highlighting the role of singular versus regular field configurations, guidance laws based on phase gradients, and the status of back-reaction between particle and wave. Recent Lagrangian and Lorentz-covariant formulations inspired by hydrodynamic pilot-wave systems will be discussed as concrete realizations of de Broglie-like dynamics, clarifying the connections and distinctions between de Broglie, Bohm, and contemporary pilot-wave models.

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Ballistic Electron Transport Described by Higher-Order Schrödinger equations

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A fourth-order Schrödinger equation for charge transport in semiconductors operating in the ballistic regime is introduced, incorporating non-parabolic effects in the dispersion relation and thereby going beyond the simple effective-mass approximation. As in the standard second-order formulation, the problem is confined to a finite spatial domain and equipped with transparent boundary conditions to simulate charge transport in a quantum coupler, where an active device region is connected to leads acting as reservoirs. Several analytical properties of the model are established, and a new expression for the current is derived. Numerical results highlight the main qualitative features of the solutions. In particular, interference effects appear due to the richer wave structure induced by the fourth-order formulation—effects absent in the effective-mass approximation. Building on this approach, a hierarchy of models is further developed, each governed by a Schrödinger equation of progressively higher order. Several analytical properties of these generalized models are analyzed and a unified current formula valid for any order is derived. Numerical simulations of a resonant tunneling diode (RTD) demonstrate the impact of the generalized formulation on device behavior.

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Long-time dynamics of open quantum systems

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In this talk, we investigate the asymptotic discrete-time evolution of open quantum systems both in the Schrödinger [1, 2] and Heisenberg [3] pictures. In particular, we present the general structure of the attractor subspace and characterize the resulting asymptotic dynamics which, remarkably, features unitary evolutions within individual superselection sectors as well as permutations among them. The latter component may be interpreted as a classical and non-Markovian contribution emerging in the long-time limit. The theoretical analysis is illustrated with several paradigmatic qubit and qutrit examples.

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Trend to equilibrium of the Wigner function describing a quantum particle subject to decoherence

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Decoherence can be broadly described as the dynamical loss of the quantum properties of a system. For this reason, the study of decoherence plays a crucial role both in fundamental physics, such as in the analysis of the quantum-to-classical transition, and in applied physics, for instance in the context of quantum computing [1]. In Ref. [2], building on the decoherence mechanism investigated in Ref. [3], the following Wigner equation with decoherence was introduced:

$$\partial_t w + p\partial_x w = \eta\partial_p(pw) - \nu\gamma * w, \tag{1}$$

where η and ν are positive constants, $*$ denotes convolution with respect to the variable p, and $\gamma(p)$ is a suitable kernel depending on the microscopic details of the collisions responsible for decoherence. The Wigner–Fokker–Planck equation, which represents a classical model of decoherence [4], is recovered as a quadratic approximation of the Fourier transform of γ.

In this communication, we consider the space-homogeneous version of Eq. (1) and show that, under suitable assumptions on the initial state and on the ratio ν/η, the solution $w(t,p)$ converges in L^2 as $t \to \infty$ to an equilibrium state exhibiting a power-law decay. This result extends, in a more general framework, a property of the Wigner–Fokker–Planck equation, whose solutions are known to converge at large times to a Gaussian distribution.

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Quantum teacher-student scenarios

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Two models of quantum perceptrons, one continuous and the other discrete will be presented together with results concerning their performances in the context classically known as teacher-student scenario, where a binary classification problem is performed by a student perceptron based on the outcomes provided by a teacher perceptron.

Also in a non-commutative scenario, interesting information can be gathered by means of the Gardner's statistical approach to learning problems.

Quantization of Fuzzy-Set-Valued Measures and the Structure of Quantum Observables

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Fuzzy structures have proven to be conceptually and mathematically relevant in quantum physics, particularly within the unsharp formalism, which addresses several limitations of the standard projection-based framework—most notably, the description of approximate joint measurements of incompatible observables. In this approach, fuzziness enters the quantum domain through unsharp (or fuzzy) observables.

This talk presents fuzzy quantum observables as the quantization of fuzzy-set-valued measures. The quantization procedure is implemented via positive operator-valued measures (POVMs). Two distinct cases are analyzed. When quantization is realized through a commutative POVM, the Łukasiewicz logic intrinsic to the structure of fuzzy sets is preserved. By contrast, quantization procedures based on non-commutative POVMs generally preserve only the weaker structure of fuzzy σ-orthoposets [1].

This is connected to the phase space representation of quantum mechanics where fuzzy sets play a crucial role.

Finally, it will be argued that fuzziness alone does not account for the genuinely distinctive features of quantum mechanics. Quantum non-classicality also fundamentally involves contextuality and non-commutativity, which go beyond graded intrinsic uncertainty. From this perspective, a systematic comparison between fuzzy and quantum frameworks provides a rigorous methodological tool for analyzing and classifying different forms of non-classicality and uncertainty, thereby clarifying both their conceptual connections and their foundational differences [2].

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Low-Temperature-Plasma Modeling with Maximum-Entropy Moment Methods

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Moment methods offer simplified solutions to kinetic equations—such as the Boltzmann or Vlasov equation—by solving for a finite set of moments instead of the full velocity distribution function. In terms of gas dynamics, the gas state represented by moment methods can be at or out of thermodynamic equilibrium. In this work, we consider the maximum-entropy family of moment methods [1], which allow to reproduce non-equilibrium features such as elongated, bi-modals, Druyvestein-like and other non-Maxwellian distributions [2]. These models are able to capture viscosity effects and a non-local heat flux via a completely hyperbolic formulation.

In particular, this work discusses the application of the fourth-order maximum-entropy method to low-temperature plasmas characteristic of Hall thrusters [3]. These devices present a crossed electric and magnetic field configuration, resulting in asymmetric ring-like distribution functions. The formulation of the model is briefly discussed and the accuracy of the model is analyzed via zero- and two-dimensional simulations. The dispersion relation associated with the maximum-entropy equations is then studied for electrostatic waves and Landau damping, and the occurrence of two-stream instabilities are discussed [4].

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Biomechanics of collective cell invasion

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Collective invasion may occur by single mesenchymal cells or hybrid epithelial-mesenchymal cell groups that detach from cancerous tissue in an early stage before cells proliferate. Tumors may also emit invading protrusions of epithelial cells. We have devised a novel hybrid cellular Potts model comprising passive and active cells able to describe these different types of collective invasion. Cell types have different adhesion properties and sensitivity to active forces. The mechanics of cells and the substrate is worked out by finite element methods and fractional step Monte Carlo dynamics. Durotaxis (motion led by stiffness gradients of the substrate) and active pushing or pulling forces have different symmetry properties and are included in different half steps of the method. Compared with a single step method, fractional step produces more realistic cellular invasion scenarios with little computational effort [1].

Besides cellular mechanics, biochemical mechanisms determine how cells acquire their different phenotypes and adopt their motion characteristics [2]. On a macroscopic supracellular scale, continuum equations describing densities of cells with different phenotype, velocities and shape tensors can be obtained by coarse graining the cellular models (see [3] for a simpler model). Overall, our work is a significant first step towards more complete descriptions of metastasis.

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Full- and low-rank exponential Euler integrators for Lindblad quantum simulations

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This talk illustrates a new computational framework [1] for high-dimensional structure-preserving low-rank approximation of the Lindblad equation as the fundamental model for open quantum systems. The focus of the talk is on novel full- and low-rank exponential Euler integrators that preserve positivity and trace unconditionally, which is essential for physical applications.

Error estimates for the two classes of exponential integration methods are presented. Results of numerical experiments are discussed that illustrate the effectiveness of the proposed schemes for simulation of high-dimensional qudits.

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Hydrodynamical model for a GaAs Resonant Tunneling Diode

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Resonant tunneling diodes (RTDs) provide a mathematically significant example of quantum transport in open semiconductor nanostructures, where quantum confinement and tunneling effects give rise to nonclassical macroscopic behavior. The device considered in this work is a double-barrier heterostructure fabricated in GaAs, forming a quantum well that supports quasi-bound states whose resonant interaction with charge carriers produces enhanced transmission and a non-monotone current-voltage characteristic.
Electron transport is well described at the kinetic level by a Wigner-Boltzmann equation for the Wigner quasi distribution function ($w(\mathbf{x}, \mathbf{k}, t)$),

$$\frac{\partial}{\partial t} w(\mathbf{x}, \mathbf{k}, t) + \frac{\hbar}{m*} \mathbf{k} \cdot \nabla_x w(\mathbf{x}, \mathbf{k}, t) + \frac{e}{\hbar} \nabla_x \phi \cdot \nabla_k w(\mathbf{x}, \mathbf{k}, t) = \mathcal{Q}(w) + \mathcal{C}(w) \tag{1}$$

where $\mathcal{Q}(w)$ denotes the quantum evolution operator, accounting for nonlocal effects associated with the underlying Schrödinger dynamics, and $\mathcal{C}(w)$ models scattering processes due to phonons and lattice impurities.
Starting from this kinetic formulation, an effective hydrodynamic model is obtained by taking suitable moments of the Wigner-Boltzmann equation. The resulting moments system is closed by means of the maximum entropy principle, yielding a consistent hydrodynamic description that retains the influence of quantum coherence and resonant states on charge density, current, and energy transport. This approach establishes a rigorous link between quantum kinetic theory, semiclassical analysis, and nonlinear hydrodynamic equations for resonant tunneling devices.

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On the derivation of the Boltzmann-Nordheim equation for weakly interacting bosons

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We investigate the behaviour of a gas of weakly interacting bosons in a box with periodic boundary conditions. Starting from a quasi-free truncation of the quantum BBGKY hierarchy and performing a suitable ultraviolet cutoff, we show that up to some explicit errors, one can recover the evolution of the initial density of the gas through the Boltzmann-Nordheim equation.

Tensor Network simulation of Multi-Emitter Waveguide QED

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Waveguide Quantum Electrodynamics (Waveguide QED) is a promising and versatile platform for studying fundamental *light-matter interactions* and *quantum technology* implementations [1]. Notably, interesting effects emerge when two or more quantum emitters are coupled to the waveguide, including *collective phenomena*, e.g., superradiance and formation of bound states in the continuum (BICs) [2, 3].
An effective approach to address the behaviour of such systems is via Tensor Network quantum-inspired simulation techniques, enabling to efficiently simulate the real-time dynamics of many-body quantum systems, i.e, a waveguide QED platform.
In particular, I will present a method based on **Matrix Product States (MPS)** to model a waveguide QED architecture featuring multiple emitter pairs and simulate its dynamics in the *non-Markovian* regime. Then, I will discuss the obtained results, focusing on the emergence of BICs and other collective effects in the long-time limit [4].

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Quantum hybrids

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This talk presents results on quantum hybrid models, namely Schrödinger and nonlinear Schrödinger equations posed on domains of mixed dimensionality, where a one–dimensional component (a lead or antenna) is coupled to a two–dimensional domain through a point interaction. We review the mathematical formulation via self-adjoint coupling conditions at the junction and discuss how the hybrid geometry affects spectral and scattering properties.

We then address the nonlinear regime, focusing on existence and qualitative properties of ground states for NLS on hybrid domains, including the hybrid plane and double-plane configurations, as well as related models with point defects in two and three dimensions. Recent developments involving hybrid structures where one of the components is a compact domain will also be discussed. Finally, we present preliminary results for the three-dimensional nonlinear Winter model, highlighting new features arising from the interplay between nonlinearity, geometry, and singular coupling.

Hyperbolic viscous flow using quaternion fields

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In this talk, I show how quaternions can be used to write a Partial Differential Equation for the evolution of the inverse deformation gradient matrix and how the properties of quaternions give rise to very benign evolution equations in fluid flow. I show results concerning applications to high Reynolds turbulent simulations and discuss the unusual behaviour of quaternion-based PDEs.

Existence of Bulk Vortices in Superconductors with Strong Magnetic Fields

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We first review the phenomenon of nucleation of vortice in a type-II superconductor immersed in an external magnetic field in the framework of the the Ginzburg-Landau theory [2, 3, 4, 5, 6, 7, 8, 9]. We discuss the phase transitions occurring in the sample as the intensity of the field grows and focus on the regime close but below the second critical field, where superconductivity survives in the bulk of the material but the magnetic field penetrates the sample. Through a two-scale vortex construction, we obtain [1] precise estimates for the vortex distribution and prove the existence of isolated defects with non-trivial winding numbers. In this respect, our work provides the first rigorous mathematical proof of the existence of isolated vortices for fields comparable to the second critical one.

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Computation and Verification of Spectra for Non-Hermitian Systems

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We establish a connection between quantum mechanics and computation, revealing fundamental limitations for algorithms computing spectra, especially in non-Hermitian settings. Introducing the concept of locally trivial pseudospectra, we show such assumptions are necessary for spectral computation. Locally trivial pseudospectra adapt dynamically to system energies, enabling spectral analysis across a broad class of challenging non-Hermitian problems. Exploiting this framework, we overcome a longstanding obstacle by computing the eigenvalues and eigenfunctions of the imaginary cubic oscillator $H_B = p^2 + ix^3$ with error bounds and no spurious modes — yielding, to our knowledge, the first such error-controlled result. We confirm, for instance, the 100th eigenvalue as 627.6947122484365113526737029011536.... Here, truncation-induced PT-symmetry breaking causes spurious eigenvalues— a pitfall our method avoids, highlighting the link between truncation and physics. Finally, we illustrate the approach's generality via spectral computations for a range of physically relevant operators. We provide a rigorous framework linking computational theory to quantum mechanics and offers a precise tool for spectral calculations with error bounds.

Effects of time-reversal asymmetry: two quantum graph examples

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We discuss effects of time-reversal invariance violation with the focus on the simplest example of the vertex coupling with this property. We address two questions. The first one concerns the maximization of the ground state eigenvalue in finite graphs with this coupling. The other is related to quantum chaos: we demonstrate an example of a graph the properties of which, both in the eigenvalue spacing distribution and the form factor, differ from what is expected in this situation. The results come from a common work with Ram Band, Divya Goel, Jonathan Rohleder and Aviya Strauss.

The quantum harmonic oscillator on a circle — fragmentation of the algebraic method

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A particle on a circle in a quadratic potential exhibits a spectrum which is not harmonic, despite having all algebraic properties of the quantum harmonic oscillator. This raises the question where the usual algebraic argument —implying integer gaps— fails. The answer is illuminating and covers a surprisingly rich range of physical phenomena for such a simple model.

Entangled Stationary States of Gaussian Open Quantum Systems

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We consider 2-mode systems with quadratic Hamiltonian in creation and annihilation operators in which each mode interacts with a reservoir. We show that the partial trace of on the 2-mode system of the stationary state of whole open system may be entangled or not according to the choice of certain parameters.

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Two-layer sharply stratified Euler fluids in three dimensions: a Hamiltonian setting

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We consider three-dimensional two layer incompressible Euler fluids from a Hamiltonian perspective. A natural Hamiltonian structure for the effective $2D$ model described by the interface-value of the field variables is obtained by means of a Hamiltonian reduction process from a suitable $3D$ Poisson structure. We consider the problem of expressing the fluid's energy in terms of the reduced variables, and show that in the weakly non linear approximation our procedure gives rise to a $2D$ Kaup-Broer-Kupershmidt Boussinesq (KBK-B) model with "critical" parameters. A model weakly dependent on the y-horizontal direction is also discussed and shown to encompass the celebrated Kadomtsev-Petviashvili (KP) equation. A Dirac-type reduction process of the Hamiltonian structure of the KBK-B model yields a natural albeit asymptotic Hamiltonian structure for KP *qua* $2+1$-dimensional model. This talk is based on [1]

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Turing Mechanisms in a Multimode Quantum System

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Turing instabilities provide a fundamental mechanism for pattern selection in classical reaction–diffusion systems, yet their extension to genuinely quantum settings remains limited and only partially explored. Existing approaches typically rely on semiclassical reductions or mean-field analogies, with only a few works addressing pattern formation in open quantum systems and limited one-two mode lattices [1, 2].

In this talk, we investigate a multimode quantum system governed by a Lindblad master equation, where local and nonlocal dissipative processes give rise to Turing-like instabilities. By employing the phase-space formulation based on the Wigner function, we show how in the semi-classical limit the quantum dynamics can be recast into an effective Fokker–Planck framework, allowing a systematic analysis of linear instabilities and mode selection.

Different classes of spatial patterns can be selectively determined depending on the intensity of the dissipators. This approach provides a framework to connect classical Turing theory with genuinely quantum effects, highlighting both its potential and its current limitations in the quantum regime.

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Spectral and angular variables for open qudit dynamics

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In this lecture, we introduce a parametrization of finite-dimensional quantum states that is particularly adapted to the study of open qudit dynamics. Any generic density matrix in $\mathbb{C}^n$ can be described by separating spectral variables, which encode the eigenvalue content, from angular variables associated with the unitary degrees of freedom. These angular variables naturally live on the coset space $\mathrm{SU}(n)/\mathbb{T}$, in the spirit of the Tilma–Sudarshan construction.

A key outcome of this parametrization is a partial decoupling of the GKLS evolution: while the angular variables are driven by both the Hamiltonian and the dissipative contributions, the spectral variables evolve under the dissipative part only. This separation provides a transparent geometric picture of decoherence and dissipation. Simple examples for qubits and qutrits are discussed, together with an alternative, geometrically motivated notion of purity expressed solely in terms of the spectral variables in a linear way.

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An operatorial View of Competition and Cooperation in a Network of Economic Agents

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A network of agents interacting both with competitive and/or cooperative mechanisms is modeled by using fermionic ladder operators [1]. The time evolution of the network is assumed to be governed by a Hermitian time–independent Hamiltonian operator, and the mean values of the number operators are interpreted as a measure of the wealth status of the agents. Besides the standard Heisenberg view, we use the recently introduced $(\mathcal{H},\rho)$–induced dynamics approach [2] to account for some actions able to provide a self–adjustment of the network according to its time evolution. Some numerical simulations are presented and discussed. Remarkably, we show that, in a network where cooperation may emerge, the average wealth of the agents is higher, and there is a very low level of inequality.

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Nonlinear evolution equation for the heat flux, non-linear heat waves and non-Newtonian phonon hydrodynamics

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We discuss two nonlinear extensions of the Maxwell-Cattaneo equation [1, 2] and of the Guyer-Krumhansl equation for the heat flux [3, 4, 5]. One of such extensions uses as independent variable the thermodynamic conjugate to the heat flux, and it leads to for the heat flux to an equation formally analogous to that for the electric field in non-linear optics, suggesting the possibility of phenomena such as frequency duplication or of Kerr effect for non-linear heat waves. When non-local terms are taken into account, such extensions leads to a version of non-Newtonian phonon hydrodynamics, comparable to a power-law model for phonon hydrodynamics [3].

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The relativistic rotated harmonic oscillator

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We introduce a relativistic version of the non-self-adjoint operator obtained by a dilation analytic transformation of the quantum harmonic oscillator. While the spectrum is real and discrete, we show that the eigenfunctions do not form a basis and that the pseudospectra are highly non-trivial.

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Geometrical Description of Quantum Mechanics: Wigner's Problem

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In a short paper, back in 1950, Wigner asked the following question:
Do Equations of Motion Determine the Commutation Relations?
By using the geometrical formulation of QM ,we shall show that the answer is negative.
For finite level quantum systems, we will show how to determine alternative commutation relations on the complex projective space (Hilbert manifold).

Existence, Localization and uniqueness of solutions of elliptic PDEs with application to population dynamics

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Abstract

We study a class of semilinear elliptic boundary value problems with nonlinear operatorial boundary conditions of the form

$$\begin{cases} Lu(x) = f(x, u(x)), & x \in \Omega, \\ u(x) = \mathcal{H}(u)(x), & x \in \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^d$ is bounded, L is a strictly elliptic second–order operator, f is a Carathéodory nonlinearity, and $\mathcal{H}$ is a nonlinear boundary operator, possibly of integral (Urysohn or Hammerstein) type.

Using elliptic regularity and the positivity and compactness properties of L^{-1}, the problem is rewritten as the fixed point equation

$$u = L^{-1}F(u) + R\mathcal{H}(u),$$

where F is the associated Nemitskii operator and R is the boundary resolution operator. The analysis is based on the fixed point index for k-set contractions in cones. This framework yields existence and localization of strong solutions, positivity results under suitable sign assumptions, eigenvalue-type criteria for nontrivial solutions, and uniqueness via the Banach–Caccioppoli contraction principle under quantitative Lipschitz conditions.

As an application, we consider a reaction–diffusion model derived from predator–prey dynamics with Holling type II functional response and predator interference. Under a quasi-stationary prey assumption, the predator density u satisfies the following stationary equation coupled with a global nonlocal boundary feedback

$$-\Delta u = \frac{Au}{B + Cu_+} - d\,u + S(x), \quad u|_{\partial\Omega} = \max\big(g(x), \alpha\,\mathcal{J}(u)\big),$$

where $\mathcal{J}(u)$ is a weighted global activity indicator.

The approach is flexible and suitable for further applications to nonlinear models with nonlocal interactions and boundary feedback mechanisms.

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Semiclassical Einstein Equations in Cosmological Spacetimes

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The semiclassical formulation of gravity is discussed in the framework of Algebraic Quantum Field Theory in Curved Spacetimes. The main topic of the talk are the Semiclassical Einstein Equations, which incorporate the backreaction of quantum matter on the geometry of classical spacetimes [1, 2].

In the first part of the talk, the general features of the equations are described with particular emphasis on their mathematical formulation within the framework of algebraic, locally covariant quantum theories, where *-algebras provide the fundamental structures encoding the quantum observables of the theory. Then, we show that the expectation value of the quantum stress-energy tensor entering the equations contains contributions with derivatives of the metric higher than the second, appearing also in a non-local form.

In the second part of the talk, the Semiclassical Einstein Equations are discussed in the framework of cosmological spacetimes. The main topic is the analysis of their solutions when the dynamics is driven by the quantum stress-energy tensor of scalar and Maxwell fields. Firstly, we prove that an initial-value problem for local solutions may be provided for a free, massive, quantum scalar field after fixing some initial data on the geometry [3]. Secondly, we show how to model stochastically the quantum fluctuations induced by thermal Maxwell fields in the perturbative linearized regime, according to Fewster and Verch's local and covariant measurement scheme [4, 5].

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Quantum dynamics in 2D systems with non-trivial topology

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Topologically non-trivial configurations in quantum systems and nanostructures lead to fundamentally interesting and technologically relevant phenomena, such as magnetic skyrmions, the quantum Hall effect, and topological insulators. In this contribution, we present a quantum phase space formalism to investigate the quantum properties—specifically the ground state, dynamics, and stability—of a class of systems displaying non-trivial spin textures. We focus on Chern topological insulators described by a Dirac-like Hamiltonian with non-zero mass, the origins of which stem from the Bogoliubov theory of superconductivity. Finally, we model the dynamical transition of a confined quantum particle between two distinct non-trivial topological phases in a 2D system.

Spatial Localization of relativistic particles and QFT: open issues and recent results

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I will present the open long standing issues about the attempts to define a causally consistent notion of spatial localization, in terms of PVMs or POVMs, for relativistic quantum particles especially in relation to rigorous QFT. I will also present and discuss some recent results on the subject by myself and collaborators.

A nonconservative kinetic framework with logistic growth for modeling the coexistence in a multi-species ecological system

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Kinetic theory frameworks are widely used for modeling stochastic interacting systems, where the evolution primarily depends on binary interactions [1]. Recently, in this framework the action of the external force field has been introduction in order to gain a more realistic picture of some phenomena [2, 3, 4]. In this work, we introduce nonconservative kinetic equations where a particular shape external force field acts on the overall system [5]. Then, this framework is used in an ecological context for modeling the evolution of a system composed of two species interacting with a prey–predator mechanism [5]. The linear stability analysis concerned with the coexistence equilibrium point is provided, and a case where a Hopf bifurcations occurs is discussed. Finally, some relevant scenarios are numerically simulated.

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Electron decoherence in a semiconductor due to electron-phonon scattering

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A quantitative analysis of the electron decoherence is performed in the case where an electron interacts with an environment consisting of a phonon bath at a given temperature. In particular we study the decoherence in terms of the entanglement formation between the electron, initially described by a Gaussian wavepacket, and the environment by using the quantum entropy called Purity, written in terms of the Wigner function. In our work we evaluate, in a rigorous fully quantum mechanical approach, the electron-phonon interaction using the Wigner-function formalism. The problem is numerically solved for by using Monte Carlo simulations [1, 2]. Results are obtained in a one-dimensional GaAs system but they can be considered indicative of a more general behavior.

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Modeling and simulation of graphene-based electronic devices through the Boltzmann transport equation

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Electron devices based on graphene have lately received a considerable interest; in fact, they could represent the ultimate miniaturization, since the active area is only one atom tick. However, the gapless dispersion relation of graphene at the Dirac points limits the possibility of using pristine graphene instead of traditional semiconductors in Field Effect Transistors (FET). For such a reason very accurate simulations are needed.

In [1] a graphene field effect transistor (GFET) has been proposed and simulated adopting a drift-diffusion model. Here, electron devices whose active area is made of monolayer graphene are simulated adopting as mathematical model the semiclassical Boltzmann transport equations (BTEs) in the bipolar case, coupled with the Poisson equation for the electric field [2]. The system is solved by means of a discontinuous Galerkin (DG) approach [3, 4] with linear elements in the spatial coordinate and constant approximation for the wave-vector space, discretized with a polar mesh. The correct physical range for the distribution function is preserved with the maximum-principle-satisfying scheme introduced in [5].

The adopted method reveals very robust and possesses a good degree of accuracy, making it particularly well suited for capturing the complex charge transport dynamics inherent to graphene-based devices. The results for suspended monolayer graphene and GFET constitute benchmark solutions for a rigorous assessment of the validity of macroscopic models, such as drift-diffusion and hydrodynamic ones.

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Spectral scheme for an energetic Fokker-Planck equation with κ-distribution steady states

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The concern of the present talk is the presentation of two efficient numerical schemes for a specific Fokker-Planck equation describing the dynamics of energetic particles occurring in thermonuclear fusion plasmas (runaway electrons for example). In the long-time limit, the velocity distribution function of these particles tends towards a (thermal) non-equilibrium κ-distribution function which is a steady-state of the considered Fokker-Planck equation. These κ-distribution functions have the particularity of being only algebraically decaying for large velocities, thus describing very well suprathermal particle populations. The aim is hence to present two efficient spectral methods for the simulation of such energetic particle dynamics. The first method will be based on rational Chebychev basis functions, rather than on Hermite basis sets which are the basis of choice for Maxwellian steady states. The second method is based on a different polynomial basis set, constructed via the Gram-Schmidt orthogonalisation process. These two new spectral schemes, specifically adapted to the here considered physical context, shall permit to cope with the long-time asymptotics without too much numerical costs. The content of this talk was published in the works [1, 2].

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Non-Markovian Open Quantum Systems and Dynamical Entropy: A Collisional Model Perspective

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Collisional models for open quantum systems will be discussed using the Alicki–Lindblad–Fannes (ALF) entropy. In particular, the focus will be on a finite-level open quantum system coupled to a classical Markov chain, which nevertheless exhibits memory effects in its reduced dynamics with no classical counterpart. The operational interpretation of the ALF entropy is neatly obtained through the GNS construction and will be discussed in relation to the activation and superactivation of memory effects.

Asymptotic behavior in a bosonic model of love affairs with $(\mathbf{H}, \boldsymbol{\rho})$–induced dynamics

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In recent years, many operatorial models for the description of complex macroscopic classical systems with applications in bioecological and socioeconomic areas have been studied [1]. In such models the dynamics, ruled by a time independent self-adjoint Hamiltonian, has been derived following the Heisenberg view. Moreover, a variant, called (H, ρ)–induced dynamics (combining the action of the Hamiltonian and specific rules), has been proposed in order to describe in a rather natural way systems for which a nontrivial and sufficiently regular asymptotic behavior is expected [2]. This approach has been mainly applied to fermionic operatorial models.

Here, we consider a nonlinear operatorial model involving bosonic ladder operators, first introduced in [3] for describing a love relationship, and enrich the model by adding some rules that in some sense may account for a change of the attitudes of the involved actors according to the evolution of the system; the main goal of this approach is that of describing a more realistic situation. The model evolution is ruled by a time-independent Hermitian Hamiltonian operator (not necessarily quadratic), and the Heisenberg equations are suitably rewritten to take into account that the infinite–dimensional Hilbert space where bosonic operators live is truncated (so that a finite number of levels are assumed for the occupation numbers). A more complex model, involving more than two actors, is also investigated. The time evolution of the observables of the system is numerically computed.

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Transport, topology, and localization in Chern insulators and Quantum (Spin) Hall systems

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The understanding of transport properties of quantum systems out of equilibrium is a crucial challenge in Mathematical Physics. A long term goal is to explain the conductivity properties of solids starting from first principles, as *e.g.* from the Schrödinger equation governing the dynamics of electrons and ionic cores.

While the general goal appears to be still far, one has now a detailed description of the transport properties of independent electrons in a periodic background, possibly including a periodic magnetic field, as *e.g.* in Chern insulators and in Quantum Hall systems. In particular, we proved the existence of a strict relation between non-vanishing Hall conductivity, non-trivial topology of the Bloch bundle corresponding to the Fermi projector, and delocalization of the electronic states, measured in terms of (de-)localization of Wannier bases [1].

Moreover, some elements of the previous picture generalize to spin transport (Quantum Spin Hall insulators) [2] and to some extent to *non-periodic* systems [3], [4]. If time permits, these recent generalizations will also be described.

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Multipartite Entanglement and Frustration

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Quantum entanglement is a nonclassical correlation between quantum systems in which the joint state cannot be factorized into independent states of the subsystems. Measurements performed on one subsystem instantaneously constrain the possible outcomes of measurements on the other, regardless of spatial separation, as dictated by the shared quantum state. Entanglement underlies key phenomena in quantum information theory, including Bell inequality violations, quantum teleportation, and quantum computation.

While the concept of bipartite entanglement is well understood, the idea of multipartite entanglement is more elusive and the subject of intense research. After a brief introduction to the topic, we investigate multipartite entanglement through the statistical properties of pure quantum states of n qubits. By analyzing the purity distribution among balanced bipartitions, we consider typical Haar states and propose a definition of maximally multipartite entangled states, whose purity is minimal for all bipartitions. We reformulate this optimization problem into a statistical mechanics problem.

For a given subsystem, purity can always be minimized by assuming an appropriate (pure) state [1]. When considering many subsystems, the requirement that purity be minimal for all of them can generate conflicts and frustration. This highlights an interesting link between frustration and multipartite entanglement [2, 3, 4, 5].

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Ballistic electron transport: the fourth order Schrödinger transient dynamics

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In this work, an advanced description of the transient dynamics [**?**] of an electron propagating in a semiconductor device operating in the ballistic regime, based on a fourth-order Schrödinger equation is presented [1]. This formulation allows to go beyond the effective mass approximation by naturally accounting for band non-parabolicity effects, which become increasingly relevant in nanoscale devices and under high-field conditions. The quantum dynamics is investigated numerically by imposing transparent boundary conditions [3], ensuring an accurate modeling of open-system propagation and preventing spurious reflections at the edges of the computational domain. The simulations show that the dynamics governed by the fourth-order equation differs significantly from the standard second-order case: in particular, the electron evolution is slower and exhibits a richer and more persistent oscillatory behavior. These results highlight the impact of higher-order terms on ballistic quantum transport and suggest that models beyond the effective mass approximation are necessary for an accurate description of transient phenomena in modern semiconductor devices.

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Regularised interacting scalar quantum field theories and convergence of the S-matrix

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We discuss the construction of self-interacting scalar quantum field theories on d-dimensional Minkowski spacetime, focusing on a class of interaction Lagrangians given by suitable functions of the scalar field. Using perturbation theory, we express interacting field observables as formal power series expansions over the free theory. Central to this construction is the time-ordered exponential of the interaction Lagrangian, the S-matrix, which itself is a power series in the coupling constant. We introduce a regularization procedure that renders the S-matrix convergent to well-defined unitary operators. This regularization involves two parameters: one controlling the suppression of high-frequency modes in the propagators, and another regulating large field contributions in the interaction Lagrangian. We analyze by abstract arguments the removal of these regularization parameters in lower-dimensional theories and for specific interaction Lagrangians. In particular, we show that for a ϕ^4 theory in three spacetime dimensions, from sequences of regularized S-matrices obtained scaling the regularization parameters to zero we can extract convergent subsequences in the weak-* topology. The asymptotic expansions of all the possible limit points of the extracted subsequences agree with perturbation theory. We also outline how these results could be extended to the four-dimensional case.

Heat and thermal trawelling wave solutions for a nonlinear Cattaneo-Vernotte equation

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Recently, a nonlinear Cattaneo–Vernotte heat equation, in which the heat conduction and the relaxation time are expressed as linear functions of the temperature, has been derived. In this paper, we analyze a nonlinear generalization of this heat equation in the case where the heat conduction and the relaxation time are expressed as polynomial functions of the temperature. In particular, we derive analytical travelling wave solutions in a rigid thermal conductor.

The variety of admissible solutions describing the propagation of thermal and heat waves, obtained by varying the degree of the polynomial expressing the temperature, is discussed, and these solutions are depicted and explained. Finally, we address the linear stability analysis of the solutions.

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The Traveling Mathematician – A Tribute to Marcello Anile

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Mathematics is often perceived as an abstract, inward-looking discipline, practiced in isolation and far removed from the world outside. Yet for many mathematicians, the opposite is true: mathematics is a passport to the world. It opens doors to countries, cultures, institutions and people, and creates a shared language that transcends national and disciplinary boundaries.

This talk is based on the book *The Traveling Mathematician* and is offered as a tribute to Marcello Anile (1948–2007), a colleague and friend who embodied the international, human and cultural dimension of mathematics like no other. The idea for such a book was first proposed by me to Marcello during a conversation in Sinaia, Romania, in 2006. He immediately embraced the idea with enthusiasm and fully recognised himself in its spirit. Tragically, his untimely death only months later prevented him from writing the book himself.

In the years that followed, I gradually realised that the only way to honour that conversation was to write the book myself—consciously and deliberately in the spirit Marcello would have chosen. *The Traveling Mathematician* thus became both a personal narrative and an intellectual homage: my own journeys, experiences and reflections, written in a voice inspired by Marcello's curiosity, generosity and joy in travel, culture and conversation.

Through a series of personal travel narratives spanning several decades and all continents, the talk illustrates how mathematics connects people across cultures, how scientific collaborations often grow out of shared meals and long conversations, and how ideas travel just as much as people do. Conferences, workshops and research projects form the formal backbone of these journeys, but it is often the informal moments that leave the deepest impression and shape lasting collaborations.

Beyond personal recollections, the talk reflects on the broader role of mathematics in society: as a quiet but indispensable driver of technological and societal progress, and as a discipline that thrives on openness, mobility and dialogue. Ultimately, this talk is a reflection on mathematics as a shared human endeavour—one that connects places, generations and cultures, and that continues to evolve through curiosity, movement and encounter.

The one-fluid extended model of superfluid helium

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There are mainly two mathematical models which well describe the peculiarities of superfluid helium-4: the two-fluid model, proposed by Landau in 1941 [1, 2], and the one-fluid extended model [3], proposed by using Extended Thermodynamics [4, 5]. The former is basically an hydrodynamical model, which sees superfluid helium as composed by two fluids (the normal viscous component and the inviscid superfluid component), instead the latter is a thermodynamical model which includes fluxes of higher order in the set of the main fields.

The aim of this talk is to show the selfconsistency of the one fluid model, as an alternative model to the usual two-fluid model, by showing the recents results especially those related to the new concepts of "heat vorticity" and "quantum heat vortices".

There is also an intrinsic two-fluid model which naturally arises from the one-fluid extended model and may be related to the vibration of two distinct modes. This intrinsic model may be compared to the Landau's two-fluid model for a direct comparison.

Finally, we also perform some numerical simulations for a direct comparison with the experiments by the Guo's group [6, 7] in heat transport in superfluid helium. The numerical experiments will regard the profiles of the heat flux (and the so-called normal and superfluid components) in 2D counterflow turbulence.

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The Knudsen layer in the heat transport beyond the Fourier law: Application to the wave propagation at nanoscale

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In agreement with the second law of Thermodynamics, a new theoretical model for the description of the heat transfer at nanoscale in a rigid body is derived.

The proposed model introduces the concept of the Knudsen layer into non-equilibrium Thermodynamics in order to better investigate how phonon-boundary scattering may influence the heat propagation at nanoscale.

This contribution, in particular, deepens the influence of the Knudsen layer on the speed of propagation of thermal waves [1].

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From spectral projections to resolvent bounds

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Anharmonic oscillators with complex polynomial potentials $L = -\partial_x^2 + x^{2a} + ix^b$ in $L^2(\mathbb{R})$ with $a, b \in \mathbb{N}$ have rich spectral properties and were studied in various contexts, in particular as a model of $\mathcal{PT}$-symmetric quantum models if b is odd. It is known that the spectral projections P_n of L have a Riesz property if $b < a - 1$ and, on the other hand, the resolvent norm of L grows super-polynomially in a large part of the right complex half-plane if $b > a - 1$.

We show that if $a - 1 < b < 2a$, then spectral projections P_n do not have a basis property, moreover, for $\sigma = [b - (a - 1)]/(1 + a)$ and $\gamma > 0$ small enough,

$$\limsup_{n\to\infty} \frac{\|P_n\|}{\exp(\gamma n^\sigma)} = \infty.$$

Proofs are based on two groups of results which are of great interest on their own: (a) relationship between behavior (growth) of the norms of projections $\|P_n\|$ and of the resolvent $\|(z-L)^{-1}\|$ outside of the spectrum $\sigma(L)$; (b) partial fraction decompositions of special meromorphic functions $1/F$ where $F(w) = \prod_{k=1}^{\infty}\left(1 + \frac{w}{a_k}\right)$, $a_{k+1} \geq a_k > 0$, $k \in \mathbb{N}$, and the generalization of the first resolvent identity.

The talk is based on a joint work with B. Mityagin (OSU, USA).

A simple mathematical model for EPR argument

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It is well known that the paper by Einstein, Podolski and Rosen (EPR) in 1935 has been of fundamental importance both in the debate on the foundations of Quantum Mechanics and in the concrete developments of quantum theory and its applications. In the seminar we briefly recall the content of the paper and then we introduce and discuss a simple mathematical model where the EPR argument can be illustrated in a quantitative and rigorous way. This is a joint work with R. Adami and L. Barletti.

Analysis of Coupled-Thermoporoelastic Materials via the Stroh-Hamiltonian Approach

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Rooted in the theory of anisotropic elasticity, the Stroh–Hamiltonian formalism offers a powerful and structured approach to modeling wave propagation and coupled field phenomena in complex media. This work presents a reversible framework for coupled anisotropic thermoporoelasticity, extending Biot's classical poroelastic theory to incorporate thermal effects. The model is derived from the Duhamel–Neumann constitutive law and consistently couples stress, pore pressure, seepage displacement, and temperature variations. By introducing a Lagrangian density and employing a Hamiltonian formalism inspired by the Stroh approach, the governing equations are recast into canonical form, highlighting intrinsic symmetries and clarifying the role of energy-conjugate variables. The reversible setting, assumed under perfectly drained and thermally relaxed conditions, eliminates dissipative effects and enables a rigorous treatment of the coupled solid–fluid–thermal response. The framework provides a structured basis for analyzing wave propagation, boundary value problems, and multiphysics interactions in anisotropic porous solids. It offers theoretical insights into the interplay of mechanical, fluid, and thermal fields, while also supporting efficient computational approaches. Potential applications include geomechanics, porous material characterization, and engineered thermo-fluid-structure systems. The results establish a foundation for future extensions that address irreversible processes such as viscous dissipation, heat conduction, and non-equilibrium fluid transport.

Implementation on GPU of a solver for the homogeneous Boltzmann transport equation

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This work is a contribution, from the computational point of view, to the simulation of the homogeneous Boltzmann equation, meant as a first step for the simulation of Graphene Field-Effect Transistors (GFETs). It takes the work in [1] and ports the implementation to the Graphic Card Unit (GPU) by means of `Cuda` extension to `C++`. It is meant as a test in a simplified context of the parallelization strategies, aiming at the the implementation of the full model [2].

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Low-noise Pauli-consistent ensemble Monte Carlo for graphene with electron–electron scattering

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Monte Carlo (MC) methods are a standard tool for charge-transport modeling in regimes where strong nonequilibrium effects and multiple scattering mechanisms limit deterministic approaches. Graphene and related two-dimensional materials are a prominent example, since transport often occurs under degenerate conditions where fermionic statistics must be treated consistently.

A fully Pauli-consistent MC scheme must enforce Pauli blocking not only during scattering events, but also during the drift (free-flight) step. An effective approach to full Pauli consistency is the NEMC method [1, 2]. When intraband electron–electron (e–e) scattering is treated at the kernel level within this framework, the event-resolved binary collision operator becomes computationally expensive, as it depends on the evolving distribution and requires explicit partner sampling, exact energy–momentum conservation, and two-particle Pauli blocking at each attempted event [3]. This limits access to the large ensemble sizes required for low MC statistical noise.

We introduce an effective-rate Pauli-consistent NEMC formulation (ER–NEMC) in which e–e interactions are proposed using a precomputed rate that represents an averaged kernel activity, while the realization of each event is still controlled dynamically by energy–momentum conservation and instantaneous Pauli blocking at the event level, enabling simulations with very large particle numbers.

Access to the low-noise regime reveals reproducible oscillatory components in ensemble-averaged time traces. We show that these oscillations are numerical in origin, arising from deterministic drift across a discretized k-space representation, with a period that scales with field strength and grid resolution. The effect persists under major changes of grid geometry, and we outline a conservative post-processing strategy that removes only the deterministic grid-locked harmonic component without modifying the underlying MC dynamics.

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Many-Agent Systems for Accelerated Consensus in Global Optimization

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We propose a kinetic-theory perspective to accelerate consensus in global optimization. Motivated by the Kaniadakis–Quarati equation for bosons [3, 4], we extend consensus-based optimization (CBO) by introducing a superlinear drift that enhances condensation effects in the supercritical regime [1, 2]. This modification amplifies the drift in a controlled manner, preventing premature collapse of the support of the distribution function. The resulting nonlinear dynamics provide a principled route to fast convergence in high-dimensional optimization landscapes.

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