Delving into Port-Hamiltonian Systems: A Case Study Approach with Multiphysics Applications

Balázs Endrész, Lea Meissner, Stanislav Beregov

ECMI Modelling Week 2024,
Catania, Italy

29.06.2024

Agenda

Delay Line Oscillator

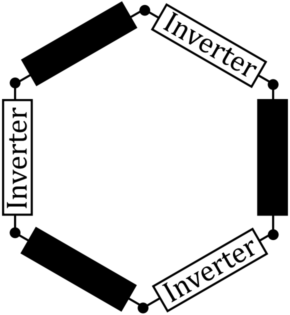

Figure: Schematic of the Delay Line Oscillator Model

CMOS Inverter

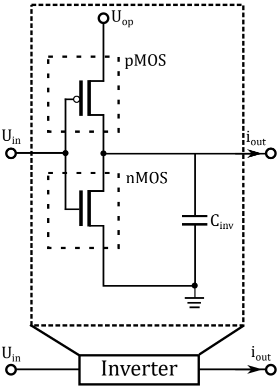

Figure: Schematic of the Inverter Model

MOS Transistor

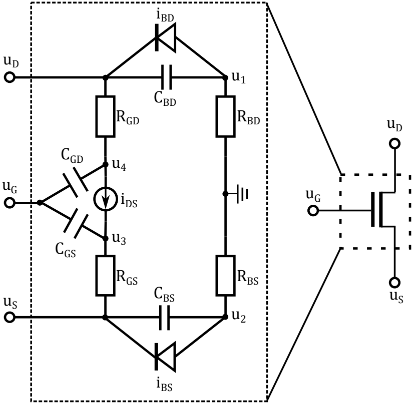

Figure: Schematic of the MOS Transistor Model

Transmission Lines

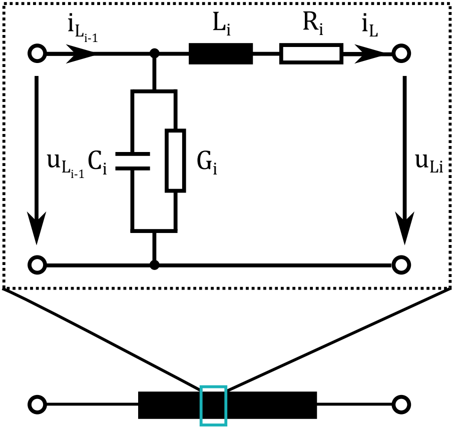

Figure: Schematic of one Segment of the Transmission Line Model

Introduction to Port-Hamiltonian Systems

Hamiltonian System:

$$\frac{d}{dt}x = J\nabla H(x), \quad x(0) = x_0$$

with

- solution $x \in \mathbb{R}^n$ of the system
- skew-symmetric matrix $J \in \mathbb{R}^{n\times n}$
- Hamiltonian $H : \mathbb{R}^n \mapsto \mathbb{R}$

Introduction to Port-Hamiltonian Systems

Adding Dissipation to the System:

$$\frac{d}{dt}x = (J - R)\,\nabla H(x) - r(\nabla H(x)), \quad x(0) = x_0$$

with

- symmetric and positive semi-definite matrix $R \in \mathbb{R}^{n \times n}$
- nonlinear accretive vector $r : \mathbb{R}^n \mapsto \mathbb{R}^n$ i.e. fulfilling $v^\top r(v) \geq 0 \forall v$

! Port-Hamiltonian Systems preserve essential physical properties such as dissipative inequalities.

Introduction to Port-Hamiltonian Systems

Coupling to the Environment:

$$\frac{d}{dt}x = (J - R)\nabla H(x) - r(\nabla H(x)) + B\,u, \quad x(0) = x_0$$
$$y = B^\top \nabla H(x)$$

with

- input $u \in \mathbb{R}^n$ of the system
- output $y \in \mathbb{R}^n$ of the system
- port-matrix $B \in \mathbb{R}^{n \times n}$

Introduction to Port-Hamiltonian Systems

Generalized to PH-DAE:

$$\frac{d}{dt} E\, x = (J - R)\, z(x) - r(z(x)) + B\, u, \quad x(0) = x_0$$
$$y = B^\top z(x)$$

with

- nonlinear mapping z of x
- a possibly singular matrix $E \in \mathbb{R}^{n \times n}$
  fulfilling the compatibility condition $E^\top z = \nabla H$

Port-Hamiltonian System of Electrical networks

$$\frac{d}{dt}\begin{pmatrix} A_c & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} q_c \\ \phi_L \\ e \\ j_v \end{pmatrix}$$

$$= \begin{pmatrix} -A_{R,l} G A_{R,l}^\top & -A_L & 0 & -A_V \\ A_L^\top & I & 0 & 0 \\ 0 & 0 & 0 & 0 \\ A_V^\top & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} e \\ \phi^{-1}\phi_L \\ q^{-1} q_c \\ j_v \end{pmatrix}$$

$$- \begin{pmatrix} A_{R,nl}\, g(e) \\ 0 \\ A_C^\top e - q^{-1} q_c \\ 0 \end{pmatrix} + \begin{pmatrix} -A_I & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -I \end{pmatrix}\begin{pmatrix} i(t) \\ v(t) \end{pmatrix}$$

Coupling of Port-Hamiltonian Systems

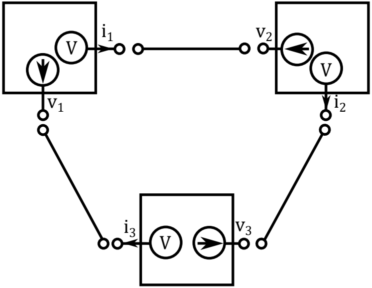

Figure: Schematic of the Coupling of Port-Hamiltonian System Model

! The overall system can be modelled as a port-Hamiltonian system too, which preserves the properties of the underlying subsystems.

Simulation of the MOS Transistor

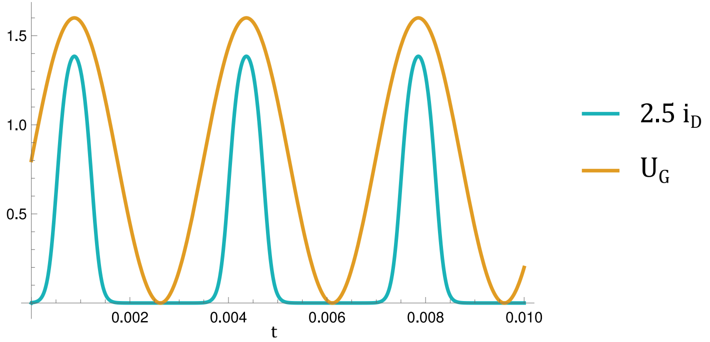

Figure: Simulated gate voltage U_D and drain current i_D for the MOS transistor with $R_{GD} = 6\,\Omega$, $R_{GS} = 0.6\,\Omega$, $R_{BD} = R_{BS} = 100\,\mathrm{M}\Omega$, $C_{GD} = C_{GS} = 0.3\,\mathrm{nF}$, $C_{BD} = 0.1\,\mathrm{nF}$, $C_{BS} = 3\,\mathrm{nF}$, $V_{th} = \pm 2.15\,\mathrm{V}$

Simulation of the CMOS Inverter

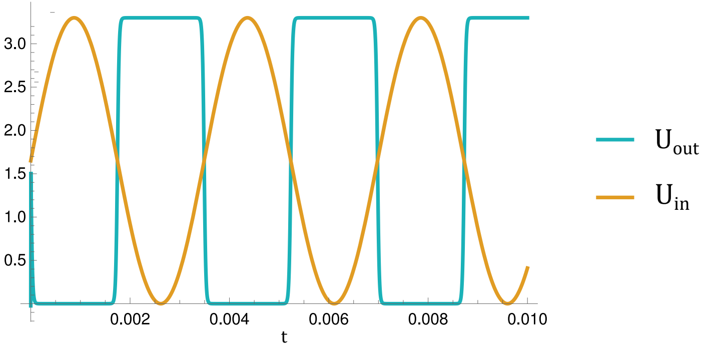

Figure: Output and input voltage of the CMOS Inverter with $C_{inv} = 1$ nF, $U_{op} = 3.3$ V.

Simulation of the CMOS Ring Oscillator

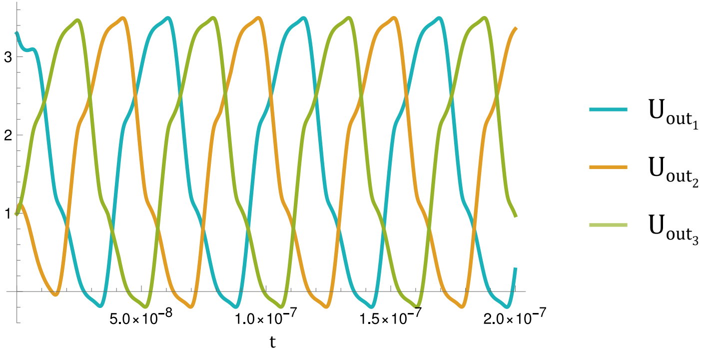

Figure: Output of the coupled inverters inside the CMOS Ring Oscillator

Simulation of the Transmission Lines

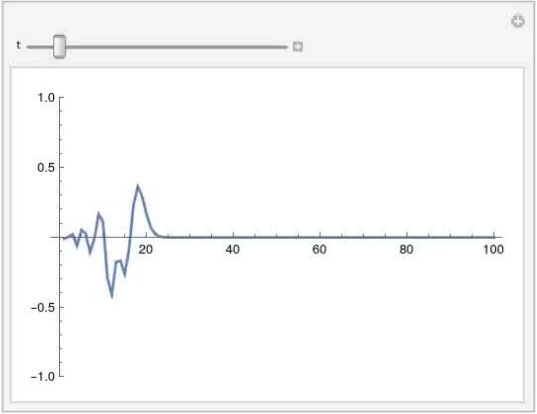

Figure: Wave propogation inside the transmission line with $G = 1\,\mathrm{mm}/\Omega, R = 1\,\mathrm{m}\Omega/\mathrm{m}, C = 1\,\mathrm{mF}/\mathrm{m}, L = 3\,\mathrm{mH}/\mathrm{m}$

Simulation of the Delay Line Oscillator

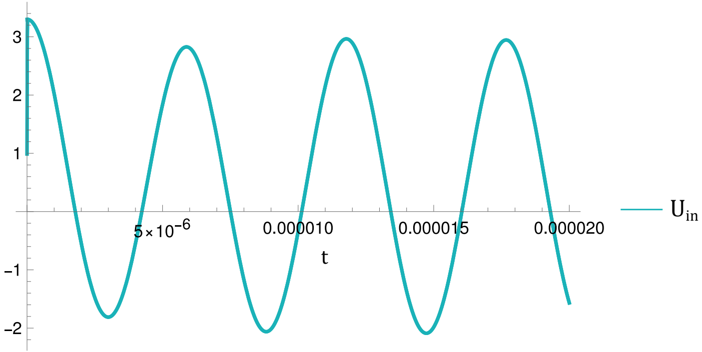

Figure: Output simulation result of the Delay Line Oscillator.

References

[1] Andreas Bartel et al. *Port-Hamiltonian Systems Modelling in Electrical Engineering*. 2023. arXiv: 2301.02024 [math.NA]. URL: https://arxiv.org/abs/2301.02024.

[2] Michael Günther et al. *Dynamic iteration schemes and port-Hamiltonian formulation in coupled DAE circuit simulation*. 2020. arXiv: 2004.12951 [math.NA]. URL: https://arxiv.org/abs/2004.12951.

[3] Michael Günther. "A joint DAE/PDE model for interconnected electrical networks". In: *Mathematical and Computer Modelling of Dynamical Systems* 6 (Aug. 2010), pp. 114–128. DOI: 10.1076/1387-3954%28200006%296%3A2%3B1-M%3BFT114.

THANK YOU
FOR YOUR ATTENTION