EUROPEAN CONSORTIUM FOR MATHEMATICS IN INDUSTRY

PROJECT

Numerical models for geothermal energy exploitation in volcanic regions

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Summary

- Physical problem
- Mathematical model
- Geothermal values
- Discretization
- Numerical results
- Conclusions and future works
- References

Physical problem

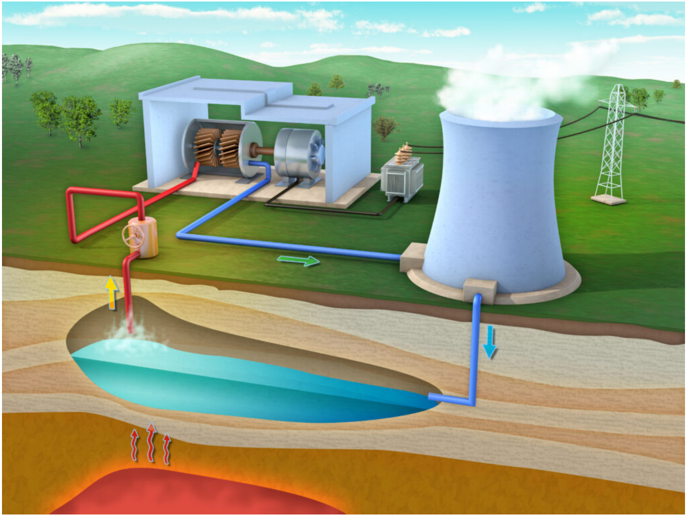

Figure: Geothermal power station

Mathematical model: equations

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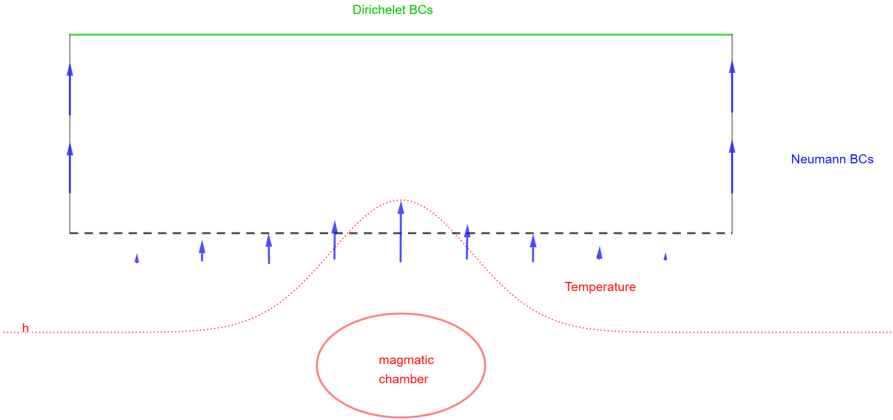
Equations

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T - \alpha \Delta T = 0 \qquad \textit{(convection-diffusion equation)}$$

$$\mathbf{u} = -\frac{k}{\mu} \nabla p \qquad \textit{Darcy Law}$$

$$div(\mathbf{u}) = 0 \qquad \textit{Conservation of mass}$$

Mathematical model: Boundary condition



Mathematical model: Temperature at $t = 0$

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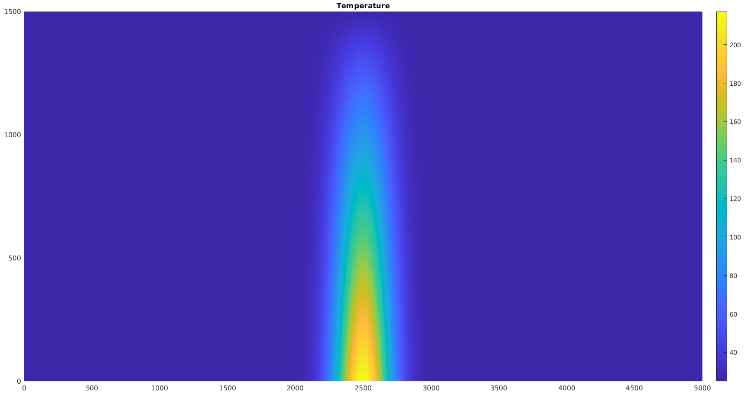

Figure: Plot of the temperature at the initial time

Geothermal values

coefficient	α	ν	μ
physical meaning	diffusivity	permeability	viscosity
units	$m^2 \cdot s^{-1}$	m^2	$Pa \cdot s^{-1}$
I scenario	$1.43 \cdot 10^{-7}$	10^{-14}	$8.9 \cdot 10^{-4}$
II scenario	$2.3 \cdot 10^{-5}$	10^{-14}	$2.2 \cdot 10^{-4}$
III scenario	$0.5 \cdot 10^{-5}$	10^{-14}	$0.6 \cdot 10^{-3}$

- Diffusivity: this constant indicates how significant the diffusion is. The greater the alpha, the more the heat spreads.
- Permeability: is the ability of underground rocks to allow fluid to pass through.
- Viscosity: in a fluid are defined as those resulting from the relative velocity of different fluid particles.

Discretization: Explicit vs Implicit

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We implemented two types of discretization for time dependance:

Equations (Forward Euler)

$$\frac{T^{n+1} - T^n}{\tau} + \mathbf{u}\nabla T^n - \alpha\Delta T^n = 0$$

Equations (Backward Euler)

$$\frac{T^{n+1} - T^n}{\tau} + \mathbf{u}\nabla T^{n+1} - \alpha\Delta T^{n+1} = 0$$

Where T^n is T evaluate at time $t_0 + n \cdot dt$.
The explicit method is easy to implement but has significant restrictions on temporal stepsize.
On the other hand, the implicit one is more robust but requires solving a linear system for each iteration.

Discretization: Advection term

1. *Central Finite Difference*, is a second-order method, but is affected by oscillation.
2. *Upwind scheme*, is a first-order method but more stable.

Equations

$$\mathbf{u} \cdot \nabla T = u_1\big(\partial_x T\big) + u_2\big(\partial_y T\big)$$

$$1.\ \partial_x T = \frac{T_{i+1,j} - T_{i-1,j}}{2\ dx} + o(||dx||^2)$$

$$2.\ \partial_x T = \begin{cases} \frac{T_{i+1,j} - T_{i,j}}{dx} & \text{if } u_1 < 0 \\ \frac{T_{i-1,j} - T_{i,j}}{dx} & \text{if } u_1 > 0 \end{cases} + o(||dx||)$$

Where $T_{ij} = T(x_i, y_j, \bar{t})$ and $\bar{t}$ is fixed. We can use *central differences* when the **Péclet** condition is satisfied: $u_1 dx < 2\alpha$.

Discretization: Diffusion Terms

We used a **second-order** scheme in space. For the Laplace operator we used classical *Central finite difference*:

$$\alpha \Delta T = \alpha\Big(\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{dx^2} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{dy^2}\Big) + o(||(dx, dy)||^2)$$

for Neumann boundary condition we used *one-sided finite difference*

$$\frac{-3T_{ij} + 4T_{ij+1} - T_{ij+2}}{2dy} = g_{\hat{n}}$$

where $g_{\hat{n}}$ is the normal component on the boundary.

Discretization: Darcy law

Since ν and μ are constant, the Darcy law and the conservation of mass read

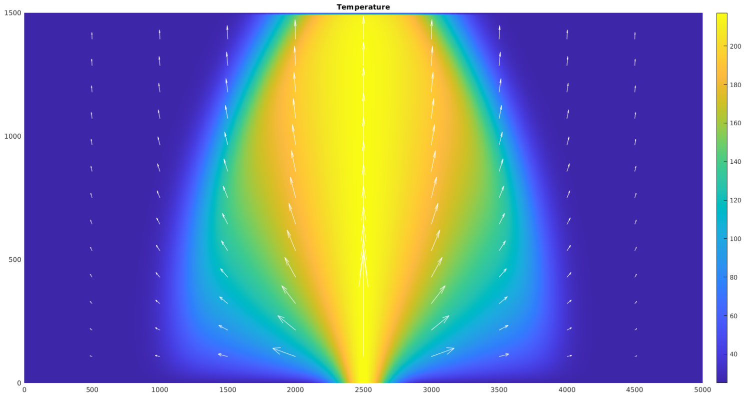
$$div(-\frac{\nu}{\mu}\nabla p) = -\frac{\nu}{\mu}div(\nabla p) = 0 \Rightarrow \Delta p = 0$$

the discretization is similar to the Laplacian of *Temperature*.
Moreover, we can approximate the velocity vector field $\mathbf{u}$ using the *central finite difference*

$$u_1 = -\frac{\nu}{\mu}\left(\frac{p_{i+1j} - p_{i-1j}}{2dx}\right) \quad u_2 = -\frac{\nu}{\mu}\left(\frac{p_{ij+1} - p_{ij-1}}{2dy}\right)$$

The velocity does not depend on time; it is a stationary flow. We solve these equations only once and then solve the heat equation.

Numerical results: III scenario after 90 days



Numerical results: Evolution

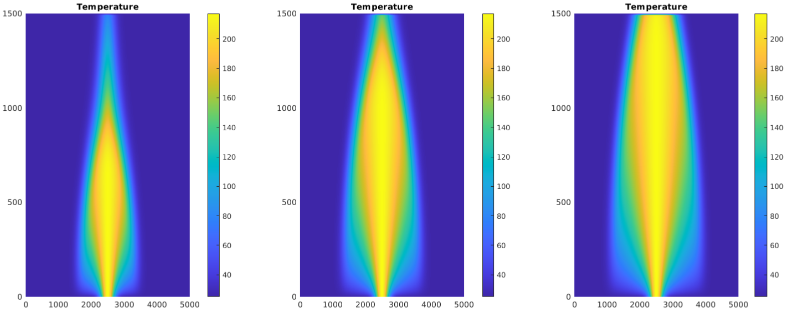
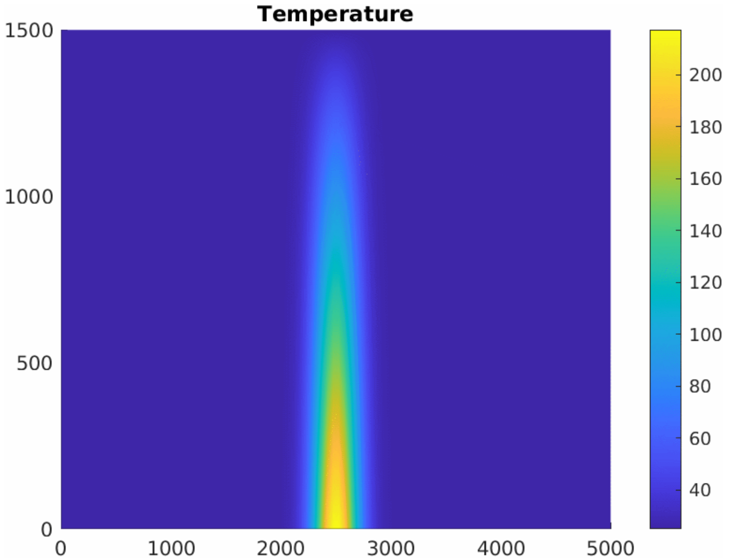
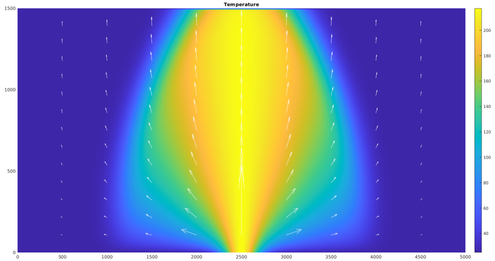


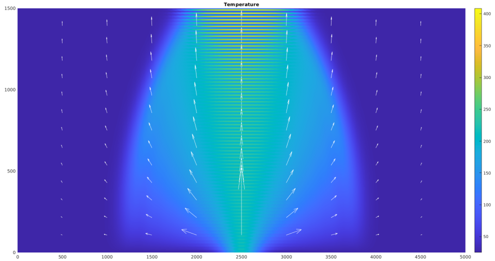
Figure: From left to right: Temperature after 30, 60, 90 days

Numerical results: Animated Evolution



Numerical result: Central F.D. vs Upwind





Conclusions and Future works

Conclusions

- $dx \neq dy$
- Upwind vs Central Finite Difference
- spreading of heat fluid
- range of depth

Future works

- $\mu = \mu(T(\mathbf{x}))$
- $\nu = \nu(\mathbf{x})$
- II order time (ex. Cranck Nicolson)

References

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Thanks for your attention!