Modelling and optimizing a semiconductor device

Participants: Giulia Elena Aliffi, Mohammed Mohammed, Andrea Panozzo
Relatori Esterno: Dr. Giovanni Nastasi & Dr. Vito Dario Camiola

Dipartimento di Matematica e Informatica - Università degli Studi di Catania

Italy

ECMI Modelling Week 2024

Catania, Italy, 23-29 June 2024

EUROPEAN CONSORTIUM FOR MATHEMATICS IN INDUSTRY

Università di Catania

Uni ct MATEMATICA E INFORMATICA

CIMAT CENTRO INTERDIPARTIMENTALE DI MATEMATICA PER LA TECNOLOGIA "A. M. ANILE"

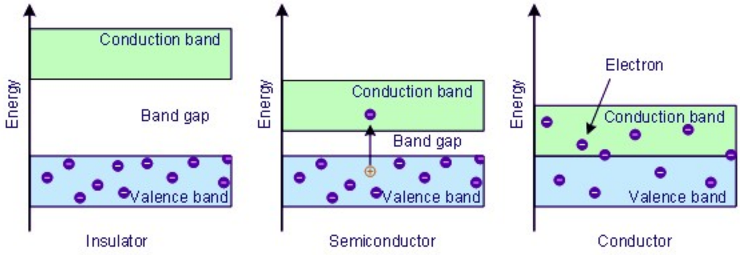
Erasmus+

associazione angelo marcello anile

Outline

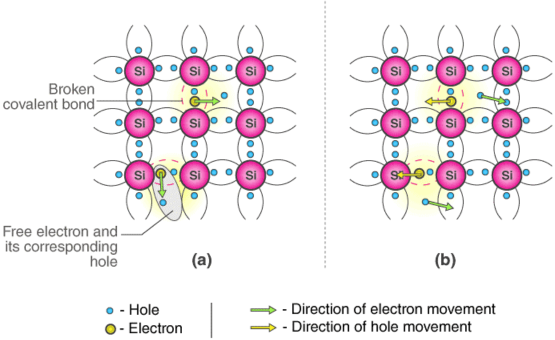
What are Semiconductors?

- Materials with electrical conductivity between conductors and insulators.
- Conduct electricity under certain conditions.
- Essential for controlling electrical current.



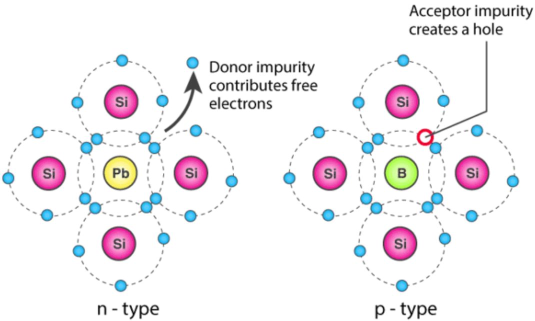
Types of Semiconductors
Intrinsic semiconductors

Pure semiconductors.



Types of Semiconductors
Extrinsic semiconductors

Doped with impurities.



N-type:

1. Mostly because of electrons.
2. Absolutely unchanged.
3. $I = I_h$ and $n_h \gg n_e$.

P type:

1. Mainly because of the holes.
2. Entirely neutral.
3. $I = I_h$ and $n_h \gg n_e$.

Applications

Transistors
What they are and their use

A MOSFET (Metal-Oxide-Semicondutor Field-Effect transistor) is a device which consistes of four terminals: Source (S), Drain (D), Gate (G) and Bulk (B). The transistor is a symmetrical structure and source and drain are interchangeable.

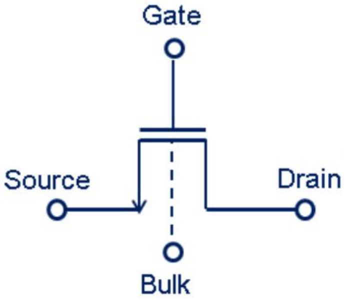


Figura: Simple view of a MOSFET design

Transistors

The source,the drain and the polysilicon (gate) terminals are heavily-doped regions and the Silicon dioxide (Si02) insulates the gate from the substrate.

Transistor operate as switches and when they are turned on, the current flows from the Source to the Drain in p-channel transistors and from Drain to Source in n-channel ones. However, carriers always travel from the source to the drain.

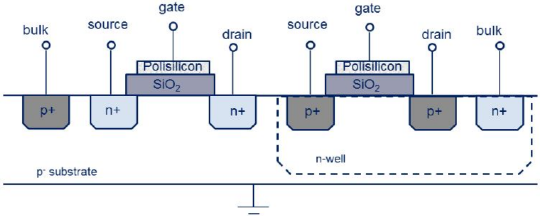


Figura: NMOS and PSON devices in CMOS technologies.

Transistors

The source,the drain and the polysilicon (gate) terminals are heavily-doped regions and the Silicon dioxide (Si02) insulates the gate from the substrate.

Transistor operate as switches and when they are turned on, the current flows from the Source to the Drain in p-channel transistors and from Drain to Source in n-channel ones. However, carriers always travel from the source to the drain.

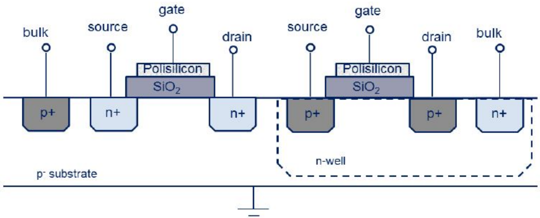


Figura: NMOS and PSON devices in CMOS technologies.

PDE associated with the transistor device

We want to model the transport of the electron carriers in our transistor (NMOS) with a drift-diffusion PDE for the electron density $n(x,t)$ coupled with the Poisson equation for the electrical potential $\phi(x)$ with $x \in \Omega$.

$$\begin{cases} \frac{\partial n}{\partial t} - \frac{1}{e}\nabla \cdot J_n &= 0 \\ -\Delta\phi + e(n - ND) &= 0 \end{cases}$$

with $J_n = e\mu_n(U_T \nabla n - n\nabla\psi)$.

$n(x,t)$ is the electron density at time $t \geq 0$ and $x \in \Omega$, e the electric charge, μ_n the electron mobility, U_T the thermal voltage, ND the doping concentration, J the electric current and $\phi(x)$ the electrical potential in $x \in \Omega$.

PDE associated with the transistor device

We want to model the transport of the electron carriers in our transistor (NMOS) with a drift-diffusion PDE for the electron density $n(x,t)$ coupled with the Poisson equation for the electrical potential $\phi(x)$ with $x \in \Omega$.

$$\begin{cases} \frac{\partial n}{\partial t} - \frac{1}{e}\nabla \cdot J_n &= 0 \\ -\Delta\phi + e(n - ND) &= 0 \end{cases}$$

with $J_n = e\mu_n(U_T \nabla n - n\nabla\psi)$.

$n(x,t)$ is the electron density at time $t \geq 0$ and $x \in \Omega$, e the electric charge, μ_n the electron mobility, U_T the thermal voltage, ND the doping concentration, J the electric current and $\phi(x)$ the electrical potential in $x \in \Omega$.

PDE associated with the transistor device

We want to model the transport of the electron carriers in our transistor (NMOS) with a drift-diffusion PDE for the electron density $n(x,t)$ coupled with the Poisson equation for the electrical potential $\phi(x)$ with $x \in \Omega$.

$$\begin{cases} \frac{\partial n}{\partial t} - \frac{1}{e}\nabla \cdot J_n &= 0 \\ -\Delta\phi + e(n - ND) &= 0 \end{cases}$$

with $J_n = e\mu_n(U_T \nabla n - n\nabla\psi)$.

$n(x,t)$ is the electron density at time $t \geq 0$ and $x \in \Omega$, e the electric charge, μ_n the electron mobility, U_T the thermal voltage, ND the doping concentration, J the electric current and $\phi(x)$ the electrical potential in $x \in \Omega$.

Finite differences

We discretized the presented equations, coupled with Dirichlet boundary conditions, using finite differences:

$$\begin{cases} \frac{n_i^{k+1} - n_i^k}{\Delta t} - \frac{1}{e} \frac{J_{i+\frac{1}{2}} - J_{i-\frac{1}{2}}}{\Delta x} = 0 & \text{Continuity equation} \\ J_{1+\frac{1}{2}} = e\mu_n \left(U_T \frac{n_{i+1}^k - n_i^k}{\Delta x} - n_{i+\frac{1}{2}}^k \frac{\phi_{i+1}^k - \phi_i^k}{\Delta x} \right) & \text{Current equation} \\ \frac{\phi_{i+1}^k - 2\phi_i^k + \phi_{i-1}^k}{2} = \frac{e}{\varepsilon} \left(n_i^k - ND \right) & \text{Poisson equation} \end{cases}$$

where we assumed the mobility $\mu_{n,i} \approx \mu(E_i) = \frac{\mu_0}{\sqrt{1 + \left(\frac{\mu_0 |E_i|}{V_S} \right)^2}}$.

Scharfetter-Gummel

To get better results we also implemented the Scharfetter-Gummel method, which has an higher order of accuracy.

To do so we introduced the slotboom variable $S(x) = n \exp\left(\frac{-\phi(x)}{U_T}\right)$, which derived, gives an expression for the current.

Integrating that in the interval $[x_i, x_{i+1}]$ we obtain the following discretization

$$J_{i+\frac{1}{2}} = e\frac{\mu_{i+\frac{1}{2}}}{h}\sigma_{i+\frac{1}{2}}\left[(n_{i+1} - n_i)\coth\left(\frac{\sigma_{i+1}}{U_T}\right) - (n_{i+1} + n_i)\right]$$

Remark: In each interval $[x_i, x_{i+1}]$ we suppose that $u(x)$ and $J(x)$ are constants while we approximate $\psi(x)$ with a piecewise linear function:

$$\psi(x) \approx \psi_i + \frac{x - x_i}{\delta x}(\psi_{i+1} - \psi_i)$$

Scharfetter-Gummel

To get better results we also implemented the Scharfetter-Gummel method, which has an higher order of accuracy.

To do so we introduced the slotboom variable $S(x) = n \exp\left(\frac{-\phi(x)}{U_T}\right)$, which derived, gives an expression for the current.

Integrating that in the interval $[x_i, x_{i+1}]$ we obtain the following discretization

$$J_{i+\frac{1}{2}} = e\frac{\mu_{i+\frac{1}{2}}}{h}\sigma_{i+\frac{1}{2}}\left[(n_{i+1} - n_i)\coth\left(\frac{\sigma_{i+1}}{U_T}\right) - (n_{i+1} + n_i)\right]$$

Remark: In each interval $[x_i, x_{i+1}]$ we suppose that $u(x)$ and $J(x)$ are constants while we approximate $\psi(x)$ with a piecewise linear function:

$$\psi(x) \approx \psi_i + \frac{x - x_i}{\delta x}(\psi_{i+1} - \psi_i)$$

Scharfetter-Gummel

To get better results we also implemented the Scharfetter-Gummel method, which has an higher order of accuracy.

To do so we introduced the slotboom variable $S(x) = n \exp\left(\frac{-\phi(x)}{U_T}\right)$, which derived, gives an expression for the current.

Integrating that in the interval $[x_i, x_{i+1}]$ we obtain the following discretization

$$J_{i+\frac{1}{2}} = e\frac{\mu_{i+\frac{1}{2}}}{h}\sigma_{i+\frac{1}{2}}\left[(n_{i+1} - n_i)\coth\left(\frac{\sigma_{i+1}}{U_T}\right) - (n_{i+1} + n_i)\right]$$

Remark: In each interval $[x_i, x_{i+1}]$ we suppose that $u(x)$ and $J(x)$ are constants while we approximate $\psi(x)$ with a piecewise linear function:

$$\psi(x) \approx \psi_i + \frac{x - x_i}{\delta x}(\psi_{i+1} - \psi_i)$$

Scharfetter-Gummel

To get better results we also implemented the Scharfetter-Gummel method, which has an higher order of accuracy.

To do so we introduced the slotboom variable $S(x) = n \exp\left(\frac{-\phi(x)}{U_T}\right)$, which derived, gives an expression for the current.

Integrating that in the interval $[x_i, x_{i+1}]$ we obtain the following discretization

$$J_{i+\frac{1}{2}} = e\frac{\mu_{i+\frac{1}{2}}}{h}\sigma_{i+\frac{1}{2}}\left[(n_{i+1} - n_i)\coth\left(\frac{\sigma_{i+1}}{U_T}\right) - (n_{i+1} + n_i)\right]$$

Remark: In each interval $[x_i, x_{i+1}]$ we suppose that $u(x)$ and $J(x)$ are constants while we approximate $\psi(x)$ with a piecewise linear function:

$$\psi(x) \approx \psi_i + \frac{x - x_i}{\delta x}(\psi_{i+1} - \psi_i)$$

Scharfetter-Gummel

To get better results we also implemented the Scharfetter-Gummel method, which has an higher order of accuracy.

To do so we introduced the slotboom variable $S(x) = n \exp\left(\frac{-\phi(x)}{U_T}\right)$, which derived, gives an expression for the current.

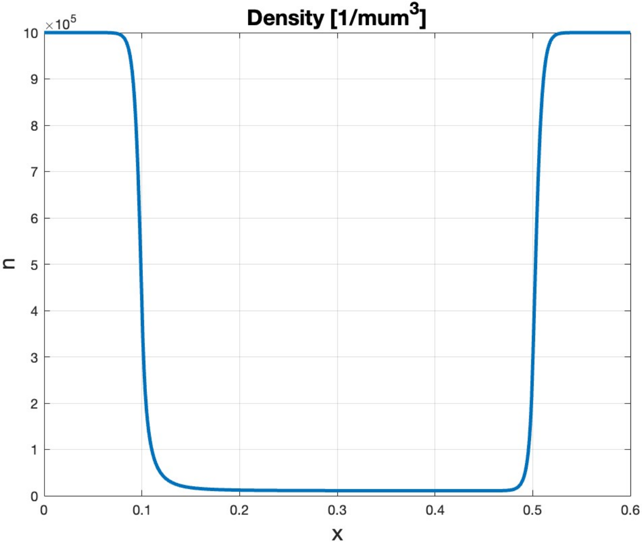
Integrating that in the interval $[x_i, x_{i+1}]$ we obtain the following discretization

$$J_{i+\frac{1}{2}} = e\frac{\mu_{i+\frac{1}{2}}}{h}\sigma_{i+\frac{1}{2}}\left[(n_{i+1} - n_i)\coth\left(\frac{\sigma_{i+1}}{U_T}\right) - (n_{i+1} + n_i)\right]$$

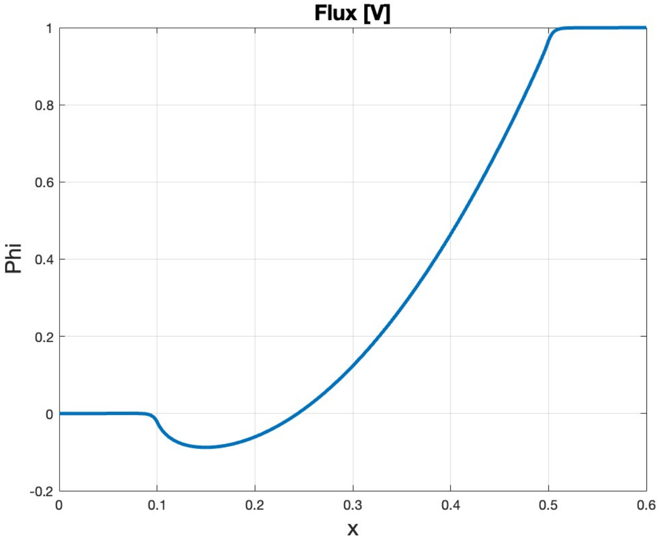
Remark: In each interval $[x_i, x_{i+1}]$ we suppose that $u(x)$ and $J(x)$ are constants while we approximate $\psi(x)$ with a piecewise linear function:

$$\psi(x) \approx \psi_i + \frac{x - x_i}{\delta x}(\psi_{i+1} - \psi_i)$$

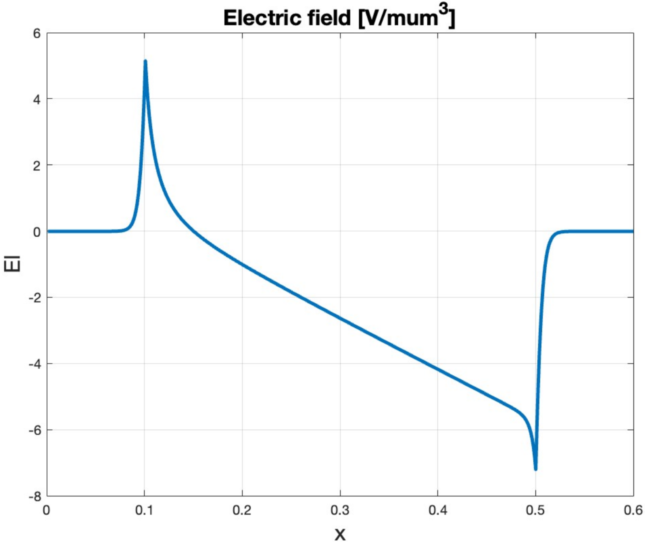
Numerical results



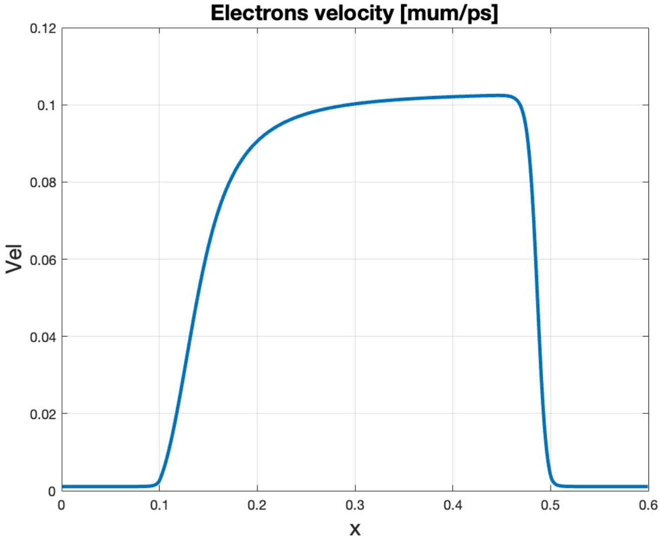
Numerical results



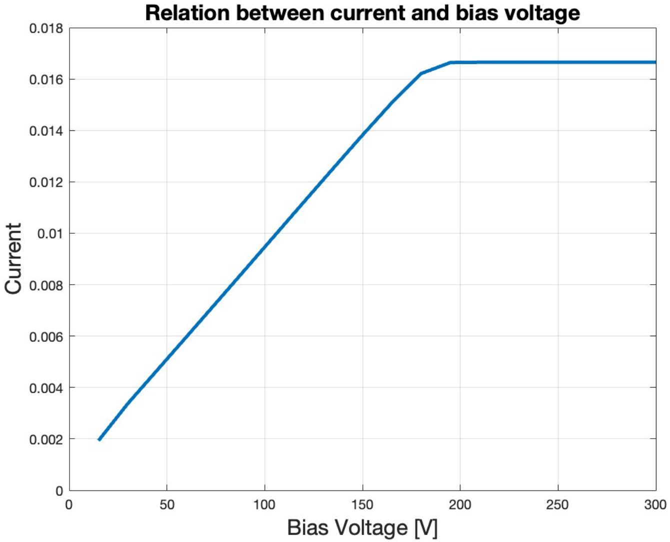
Numerical results



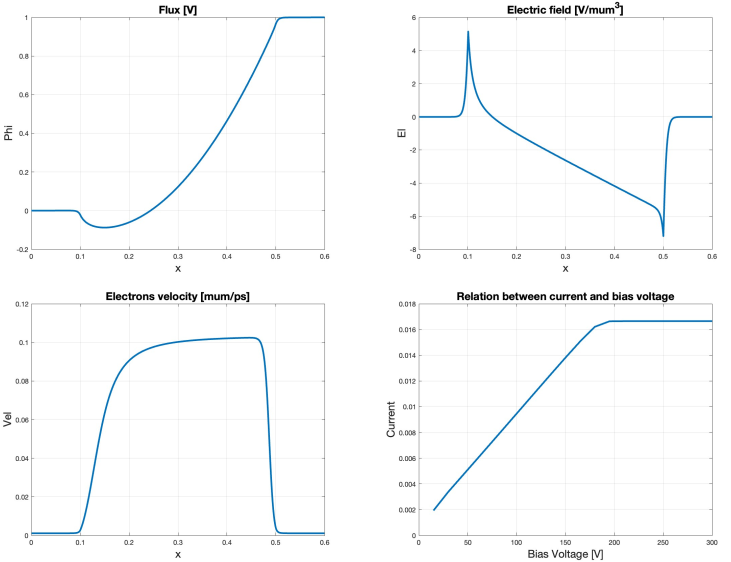
Numerical results



Numerical results



Numerical results: 1D case



Thank you for your attention!